Solving Equations

Solving Equations in the form of ax=b

In equations of the form $\boldsymbol{ax = b}$ (read as "a times x equals b) x is a variable which represents an unknown number and a and b are constants.

EXAMPLES: $ax = b$

$3x = 15$

$-4x = -16$

To isolate x, simply divide by its coefficient a.

EXAMPLE: Solve:

$$ax = b$$
$$\frac{ax}{a} = \frac{b}{a}$$
$$x = \frac{b}{a}$$

$$3x = 15$$
$$\frac{3x}{3} = \frac{15}{3}$$
$$x = 5$$

$$-4x = -16$$
$$\frac{-4x}{-4} = \frac{-16}{-4}$$
$$x = 4$$

Solving Equations in the form of x+a=b

In equations of the form $\boldsymbol{x+a=b}$ (read as "x plus a equals b) x is a variable which represents an unknown number and a and b are constants.

EXAMPLES: $x+a=b$
$x+2=7$
$x-3=4$

EXAMPLE: Solve:

$$x+a=b$$
$$x+a-a=b-a$$
$$x=b-a$$

$$x+2=7$$
$$x+2-2=7-2$$
$$x=5$$

$$x-3=4$$
$$x-3+3=4+3$$
$$x=7$$

Solving Equations in the Form ax + b = c

In equations of the form $ax + b = c$ (read as "*a* times *x* plus *b* equals *c*"), *x* is a variable which represents an unknown quantity and *a*, *b* and *c* are constants.

EXAMPLES: $ax + b = c$

$3x + 4 = 10$

$-5y - 12 = 18$

$\frac{3}{4}m + 2 = 3$

Our goal in solving these equations is to simplify the equation to the point where we have a variable equal to a constant.These equations will require us to use both the Addition Property of Equations and the Multiplication Property of Equations.

EXAMPLE: Solve:

$$3x + 4 = 10$$

$$3x + 4 - 4 = 10 - 4$$
$$3x = 6$$
$$\frac{3}{3}x = \frac{6}{3}$$
$$x = 2$$

CHECK: $3x + 4 = 10$; x=2

$$3(2) + 4 = 10$$
$$6 + 4 = 10$$
$$10 = 10$$

EXAMPLE: Solve:

$$\frac{3}{4}m + 2 = 3$$
$$\frac{3}{4}m + 2 - 2 = 3 - 2$$
$$\frac{3}{4}m = 1$$
$$\frac{4}{3} * \frac{3}{4}m = 1 * \frac{4}{3}$$
$$\frac{12}{12}m = \frac{4}{3}$$
$$m = \frac{4}{3}$$

Solving Equations in the Form $ax + b = cx + d$

In equations in the form $ax + b = cx + d$, ax and cx are variable terms and b and d are constants.

EXAMPLES: $\underline{ax + b = cx + d}$

$6x + 2 = x + 17$

$8y = 3y + 20$

$n - 2 = -3n + 6$

NOTE that $8y = 3y + 20$ still fits the form as $8y$ could be written as $8y + 0 = 3y + 20$.

Our goal in solving these equations is to simplify the equation to the point where we have a variable equal to a constant.

These equations will require us to use both the Addition Property of Equations and the Multiplication Property of Equations.

EXAMPLE: Solve: $6x + 2 = x + 17$

We must first get the variable terms on the same side of the equation.

$$-x + 6x + 2 = -x + x + 17$$

$$5x + 2 = 17$$

$$5x + 2 + (-2) = 17 + (-2)$$

$$5x = 15$$

$$\frac{5x}{5} = \frac{15}{5}$$

$$x = 3$$

Check:

$$6(3) + 2 = 3 + 17$$

$$20 = 20$$

SOLVE:

$$8y = 3y + 20$$

$$8y + (-3y) = -3y + 3y + 20$$

$$5y = 20$$
$$y = 4$$

EXAMPLE:

$$n - 2 = -3n + 6$$
$$3n + n - 2 = -3n + 3n + 6$$
$$4n - 2 = 6$$
$$4n - 2 + 2 = 6 + 2$$
$$4n = 8$$
$$1n = 2$$
$$n = 2$$

CHECK:

$$n - 2 = -3n + 6$$
$$2 - 2 = -3(2) + 6$$
$$0 = -6 + 6$$
$$0 = 0$$

NOTE that in some equations you must combine like terms before you begin to solve.

$$3x + 4 - 5x = 2 - 4x$$

$$-5x + 3x + 4 = 2 - 4x$$

$$\underbrace{-2x} + 4 = 2 - 4x$$

Now this is in the *ax* + *b* = *cx* + *d* form.
Can you finish it? The solution is −1.

EXERCISES: Solve and Check.

1. $9x - 10 = 3x + 2$
2. $-5y - 3 = 2y + 18$
3. $4x - 2 = -16 - 3x$
4. $-10a + 4 = -a - 14$
5. $6x - 1 = 2x + 2$
6. $5a + 7 = 2a + 7$
7. $3 - 2x = 15 + 4x$
8. $8y - 2 = 4y - 5$
9. $5 - 7a = 2 - 6a$
10. $10y - 3 = 3y - 2$

KEY:

1. $x = 2$
2. $a = 0$
3. $y = -3$
4. $x = -2$
5. $x = -2$
6. $y = -\frac{3}{4}$
7. $a = 2$
8. $a = 3$
9. $x = \frac{3}{4}$
10. $y = \frac{1}{7}$