TSC | WILLIAM D. LAW, JR. LEARNING COMMONS

Simplifying Variable Expressions Using the Distributive Property

We know that if we have a problem like 3(4 + 7), the Order of Operations Agreement tells us to simplify inside the grouping symbol first and then multiply.

$$3(4 + 7) = 3(11) = 33$$

However, if we change the problem to include a variable it is no longer possible to follow the Order of Operations Agreement since we cannot combine unlike terms.

EXAMPLE: $3(4x + 7)$

We cannot add inside the grouping symbol because $4x$ and 7 are not like terms. The only way we can remove the parentheses is by using the Distributive Property. To do this we must multiply both terms inside the parentheses by the term on the outside.

$$3(4x + 7) = 3(4x) + 3(7) = 12x + 21$$

EXAMPLE: $-2(m - 5)$

$-2(m-5) = -2(\text{m}) - (-2)(5)$ Multiply

$= -2m - (-10)$ Rewrite Subtraction

NOTE that when there is a negative sign in front of the parentheses and no other number, it is understood to mean -1.

$-(2x-4)$ means $-1(2x-4)$

Also note that the sometimes the number on the outside is on the right-hand side of the parentheses

$$(2y-6)(2) = (2y)(2) - (6)(2)$$

Sometimes it is necessary to simplify expressions with more than one set of grouping symbols. This will often require using the Distributive Property more than once.

EXAMPLE: $5(x - 2) - 3(x + 7)$

$$\begin{aligned} 5(x-2) - 3(x+7) &= 5(x) - 5(2) - 3(x) + (-3)(7) \\ &= 5x - 10 - 3x + (-21) \\ &= 5x - 10 - 3x - 21 \\ &= 5x - 3x - 10 - 21 \\ &= 2x - 31 \end{aligned}$$

NOTE that the subtraction sign in front of the 3 makes the 3 negative. We know this is true because if we rewrite "−3" we would get "+ (−3)." Therefore, we multiply by −3.

EXAMPLE: $2(3x - 2y) - 4(-5x + 8y)$

$$\begin{aligned} 2\,(3x-2y) - 4(\,-5x+\,8y) &= 2\,(3x) - 2(2\,y) - 4(-5x) + (-4)\,(8y) \\ &= 6x - 4y + 20x + (-32y) \\ &= 6x - 4y + 20x - 32y \\ &= 6x + 20x - 4y - 32y \\ &= 26x - 36y \end{aligned}$$

EXAMPLE: $5x - 2[4x - 3(2x + 1)]$

We will need to simplify inside the bracket first. This problem differs from the last two in that the grouping symbols are nested one inside the other.

$$5x - 2[4x - 3(2x + 1)] = 5x - 2[4x - 6x - 3]$$

Next we will combine like terms inside the bracket.

$$5x - 2[4x - 6x - 3]\ 5 = x - 2[\,-2x - 3]$$

Now we need to use the Distributive Property again.

$$5x - 2[\,-2x - 3] = 5x + 4x + 6$$

Combine like terms to get

$$= 9x + 6$$

If we put these steps together, we have the following:

$$\begin{aligned} &5x - 2[4x - 3(2x + 1)] \\ &= 5x - 2[4x - 6x - 3] \\ &= 5x - 2[\,-2x - 3] \\ &= 5x + 4x + 6 \\ &= 9x + 6 \end{aligned}$$

EXERCISES:

1. $-4(x + 7)$
2. $-(m - 4)$
3. $-2(\,-3y + 5)$
4. $8\,(3y^2 - 2)$
5. $3(x + 3) - 2(x + 4)$
6. $-2(x - y) + 5(2x - 3y)$
7. $-3[2 + 3(x + 2)]$
8. $5\,[\,2m - 4(8m - 5)]$

KEY:

1. $-4x - 28$
2. $-m + 4$
3. $6y - 10$
4. $24y^2 - 16$
5. $x + 1$
6. $8x - 13y$
7. $-9x - 24$
8. $-150m + 100$