

Writing Equivalent Fractions

A. $\frac{3}{4}$ of rectangle A is shaded.



B. $\frac{6}{8}$ of rectangle B is shaded.



It is easy to see that the shaded parts of these rectangles are the same; therefore, $\frac{3}{4}$ and $\frac{6}{8}$ are two ways of writing the same value. We say that $\frac{3}{4}$ and $\frac{6}{8}$ are **equivalent fractions**.

To **build** equivalent fractions means to write a fraction equivalent to the original fraction. The new fraction's numerator and denominator will be greater than those of the original fraction. We do this by using The Multiplication Property of One (Locate this property in your text).

$$\frac{3}{4} * 1 = \frac{3}{4}$$

Also consider that there are multiple ways to write 1 as a fraction.

$$1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \dots$$

Note: Any number divided by itself is equal to one.

Therefore, to write an equivalent fraction, remembering that any number over itself is one and that any number multiplied by one is simply that number, we can do the following.

$\frac{2}{3}$ with a denominator of 6 is....?

$$3 * ? = 6$$

$$? = \frac{6}{3}$$

$$? = 2$$

$$\text{Therefore: } \frac{2}{3} * \frac{2}{2} = \frac{4}{6}$$

II. Simplest Form of Fractions

When we write $\frac{6}{8} = \frac{3}{4}$, we are writing the value of the fraction in its simplest form. This is **sometimes** called reducing the fraction, although **simplifying** is a better description.

Look at the prime factorizations of 6 and 8.

$$\frac{6}{8} = \frac{2 \cdot 3}{2 \cdot 2 \cdot 2}$$

We see a common factor of 2 in the numerator and denominator. Since $\frac{2}{2} = 1$, we can write

$$\frac{6}{8} = \frac{2}{2} * \frac{3}{2 \cdot 2} = 1 * \frac{3}{2 \cdot 2} = \frac{3}{2 \cdot 2} = \frac{3}{4}$$

A fraction is in its simplest form when the only common factor of the numerator and denominator is 1.

(Rhetorical question - isn't 1 always a common factor of two numbers?) When we say there is no common factor, we mean no factor other than 1 is in both numbers.

We should always write fraction answers in simplest form. This means:

1. No common factor (other than 1) in the numerator and denominator.
2. Improper fractions should be written as mixed numbers.

NOTICE that either step can be done first.

Example: Fully simplify: $\frac{18}{8}$

Step 1 then 2: 1. $\frac{18}{8} = \frac{3 \cdot 3 \cdot 2}{2 \cdot 2 \cdot 2} = \frac{3 \cdot 3}{2 \cdot 2} = \frac{9}{4}$

2. $\frac{9}{4} = \frac{8}{4} + \frac{1}{4} = 2 + \frac{1}{4} = 2\frac{1}{4}$

Step 2 then 1: 1. $\frac{18}{8} = \frac{16}{8} + \frac{2}{8} = 2 + \frac{2}{8} = 2\frac{2}{8}$

2. $\frac{2}{8} = \frac{2}{2 \cdot 2 \cdot 2} = \frac{2}{2} * \frac{1}{2 \cdot 2} = \frac{1}{4}$

$$2\frac{1}{4}$$