

Least Common Multiple

Objective: Find the Least Common Multiple (LCM) of two or more numbers

Important Ideas:

1. The Least Common Multiple (LCM) of two or more numbers is the smallest number which is a multiple of all the numbers being considered.
2. Multiples of a number are the product of that number and the whole numbers 1, 2, 3, 4, 5, ...

$$1 \cdot 4 = 4$$

$$2 \cdot 4 = 8$$

$$3 \cdot 4 = 12$$

$$4 \cdot 4 = 16$$

The numbers 4, 8, 12 and 16 are the first four multiples of 4.

3. When finding the Least Common Denominator of two or more fractions, you are finding the LCM of those denominators.
4. Another way of looking at the LCM is that it is the smallest number that all of the numbers being considered will divide into.

Finding the Least Common Multiples (LCM)

To find the LCM of two or more numbers follow these steps:

1. Find the prime factorization of each number.
2. Identify all of the different prime factors which occur in each of the prime factorizations.
3. Organize the factors in a chart. (see examples)
4. Circle the largest product in each column.
5. The LCM is the product of all of the circled factors.

We will now work through several examples following these steps.

Example 1 Find the LCM of 18 and 24

The prime factorization of 18 is $2 \cdot 3 \cdot 3$

The prime factorization of 24 is $2 \cdot 2 \cdot 2 \cdot 3$

Note: See assignment 1 if you do not remember how to find the prime factorization of a number.

The different factors which occur in 18 and 24 are the prime numbers 2 and 3.

We will now organize the factors in a chart and circle the largest product in each column.

	2	3
18 =	2	3 · 3
24 =	2 · 2 · 2	3

The LCM is the product of the circled factors.

The LCM of 18 and 24 = $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

$$\text{LCM} = 72$$

This means that 72 is the smallest number that is a multiple of 18 and 24.

This also means that 72 is the smallest number that 18 and 24 will both divide into.

Example 2: Find the LCM of 14, 49 and 28.

The prime factorization of 14 is $2 \cdot 7$

The prime factorization of 49 is $7 \cdot 7$

The prime factorization of 28 is $2 \cdot 2 \cdot 7$

The different prime factors which occur in 14, 49 and 28 are 2 and 7.

We will now organize the factors in a chart and circle the largest product in each column.

	2	7
14=	2	7
49=		$7 \cdot 7$
28=	$2 \cdot 2$	7

The LCM is the product of the circled factors.

$$\text{The LCM} = 2 \cdot 2 \cdot 7 \cdot 7 = 196$$

This means that 196 is the smallest number that is a multiple of 14, 49 and 28.

This also means that 196 is the smallest number that 14, 49 and 28 will both divide into.

Example 3: Find the LCM of 15, 45 and 120

The prime factorization of 15 is $3 \cdot 5$

The prime factorization of 45 is $3 \cdot 3 \cdot 5$

The prime factorization of 120 is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

The different prime factors of 15, 45 and 120 are 2, 3 and 5.

We will now organize the factors in a chart and circle the largest product in each column.

Note that as 5 occurs only once in each prime factorization, it does not matter which of the 5's we circle.

	2	3	5
15 =		3	5
45 =		3 · 3	5
120 =	2 · 2 · 2	3	5

The LCM of 15, 45 and 120 is the product of the circled factors.

The LCM = $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 360$

This means that 360 is the smallest number which is a multiple of 15, 45 and 120.

This also means that 360 is the smallest number that 15, 45 and 120 will all divide into.

Example 4: Find the LCM of 17 and 6

17 is a prime number.

The prime factorization of 6 is $2 \cdot 3$

The different prime factors are 2, 3 and 17.

We will now organize the factors in a chart and circle the largest product in each column.

Note that each factor occurs only once.

	2	3	17
6 =	2	3	
17 =			17

The LCM of 6 and 17 is the product of the circled factors.

$$\text{The LCM} = 2 \cdot 3 \cdot 17 = 102$$

Note that 6 and 17 do not have any prime factors in common. This means that the LCM will be the product of the numbers themselves. (6 and 17)

102 is the smallest number which is a multiple of 6 and 17.

102 is the smallest number that both 6 and 17 will divide into.

Practice Exercises

Find the Least Common Multiple of the following sets of numbers.

- 9 and 15 5. 12, 15 and 45 9. 9, 14 and 28
- 16 and 246. 8, 18 and 24 10. 7, 11 and 12
- 36 and 487. 15, 20 and 30
- 19 and 158. 60, 90 and 144

Answers to Practice Problems

- LCM = 45
- LCM = 48
- LCM = 144
- LCM = 285
- LCM = 180
- LCM = 72
- LCM = 60
- LCM = 720

9. $\text{LCM} = 252$

10. $\text{LCM} = 924$