

Greatest Common Factor

Objective: Find the Greatest Common Factor (GCF) of two or more numbers Important

Ideas:

1. The Greatest Common Factor (GCF) of two or more numbers is the biggest number which is a factor of all of the numbers being considered.
2. A factor of a number is a number which will divide evenly into that number.
3. Another way of thinking of the GCF is that it is the biggest number which will divide into all the numbers being considered
4. It is easy to confuse the LCM and GCF.

The LCM of 8 and 12 is the smallest number that 8 and 12 will both divide into.
This number is 24.

The GCF of 8 and 12 is the biggest number that will divide into both 8 and 12.
This number is 4.

5. The LCM can never be smaller than the largest of the numbers being considered, while the GCF can never be larger than the smallest of the numbers being considered.

Finding the Greatest Common Factor (GCF)

To find the GCF of two or more numbers, follow these steps:

1. Find the prime factorization of each number.
2. Identify all of the different prime factors which occur in each of the prime factorizations.
3. Organize the factors in a chart. (see examples)
4. Circle the smallest number (or product) in each column that does not have a blank space.
5. The GCF is the product of all of the circled factors.

Note that steps 1–3 are the same as for finding the LCM.

We will now work through several examples following these steps.

Example 1 Find the GCF of 18 and 24

The prime factorization of 18 is $2 \cdot 3 \cdot 3$

The prime factorization of 24 is $2 \cdot 2 \cdot 2 \cdot 3$

The different factors which occur in 18 and 24 are the prime numbers 2 and 3.

We will now organize the factors in a chart and circle the smallest number in each column.

| | | |
|------|-----------|-------|
| | 2 | 3 |
| 18 = | 2 | 3 · 3 |
| 24 = | 2 · 2 · 2 | 3 |

The GCF is the product of the circled factors. The GCF of 18 and 24 = $2 \cdot 3 = 6$

This means that 6 is the biggest number which is a factor of both 18 and 24.

This also means that 6 is the biggest number which will divide into both 18 and 24.

Example 2: Find the GCF of 14, 49 and 28.

The prime factorization of 14 is $2 \cdot 7$

The prime factorization of 49 is $7 \cdot 7$

The prime factorization of 28 is $2 \cdot 2 \cdot 7$

The different prime factors which occur in 14, 49 and 28 are 2 and 7.

We will now organize the factors in a chart and circle the smallest number in each column that does not have a blank space.

| | | |
|------|-------|-------|
| | 2 | 7 |
| 14 = | 2 | 7 |
| 49 = | | 7 · 7 |
| 28 = | 2 · 2 | 7 |

The GCF is the circled factor 7.

Note that the column for 2 contains a blank space.

7 is the biggest number which is a factor of 14, 49 and 28.

7 is also the biggest number which will divide into 14, 49 and 28.

Example 3: Find the GCF of 15, 45 and 120

The prime factorization of 15 is $3 \cdot 5$

The prime factorization of 45 is $3 \cdot 3 \cdot 5$

The prime factorization of 120 is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

The different prime factors of 15, 45 and 120 are 2, 3 and 5.

We will now organize the factors in a chart and circle the smallest number in each column that does not have a blank space.

| | 2 | 3 | 5 |
|-------|---------------------|-------------|---|
| 15 = | | 3 | 5 |
| 45 = | | $3 \cdot 3$ | 5 |
| 120 = | $2 \cdot 2 \cdot 2$ | 3 | 5 |

The GCF is the product of the circled factors. The GCF of 15, 45 and 120 = $3 \cdot 5 = 15$

Note that the column for 2 contains a blank space.

15 is the biggest number that is a factor of 15, 45 and 120.

15 is also the biggest number which will divide into 15, 45 and 120.

Example 4: Find the GCF of 6 and 17

17 is a prime number.

The prime factorization of 6 is $2 \cdot 3$

The different prime factors are 2, 3 and 17.

We will now organize the factors in a chart and circle the smallest number in each column that does not have a blank space.

| | | | |
|------|---|---|----|
| 6 = | 2 | 3 | 17 |
| 17 = | 2 | 3 | |
| | | | 17 |

Note that, as each column contains a blank space, none of the factors were circled. This means that the only number that will divide evenly into 6 and 17 is the number 1.

The GCF of 6 and 17 = 1.

This means that 1 is the biggest number that is a factor of 6 and 17.

This also means that 1 is the biggest number that will divide into 6 and 17.

Take a minute before you do the practice exercises and go back and compare the LCM's with the GCF's for the sets of numbers we have been working with.

Practice Exercises

Find the Greatest Common Factors (GCF) of the following sets of numbers.

1. 9 and 15
2. 16 and 246.
3. 36 and 487.
4. 19 and 15
5. 12, 15 and 45
6. 8, 18 and 24
7. 15, 20 and 30
8. 60, 90 and 144
9. 9, 14 and 28
10. 7, 11 and 12

Answers to Practice Problems

1. 3
2. 8
3. 12
4. 1
5. 3
6. 2
7. 5
8. 6
9. 1
10. 1