

Using the Product Rule to find the Derivative

The Process for the Product Rule:

1. Given: $H(x) = f(x) \cdot g(x)$; then
2. Identify $f(x)$, $f'(x)$, $g(x)$, and $g'(x)$;
3. $H'(x) = \frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$; Product rule
4. Substitute and simplify

Some other helpful rules:

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}[cf(x)] = c f'(x)$$

$$\frac{d}{dx}(x^n) = nx^{n-1} dx$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx} \ln|x| = \frac{1}{x} dx$$

$$\frac{d}{dx}(a^x) = a^x \ln(a)$$

$$\frac{d}{dx}(\sin x) = \cos x dx$$

$$\frac{d}{dx}(\cos x) = -\sin x dx$$

Algebraic Example of Product Rule:

$$\text{Given: } H(x) = (x^2 + 1)(x^3 + 6)^7$$

$$f(x) = x^2 + 1$$

$$f'(x) = 2x$$

$$g(x) = (x^3 + 6)^7$$

$$g'(x) = 7(x^3 + 6)^6(3x^2) = 21x^2(x^3 + 6)^6; \text{ using the chain rule}$$

$$H'(x) = f(x)g'(x) + g(x)f'(x) \text{ so, we have:}$$

$$H'(x) = (x^2 + 1)[21x^2(x^3 + 6)^6] + (x^3 + 6)^7(2x); \text{ ask your teacher if you can stop here!}$$

Factor out like terms and then continue to simplify.

$$H'(x) = (x^3 + 6)^6 x[(x^2 + 1)(21x) + (x^3 + 6)(2)]; \text{ distribute}$$

$$H'(x) = (x^3 + 6)^6 x[21x^3 + 21x + 2x^3 + 12]; \text{ combine like terms}$$

$$H'(x) = x(x^3 + 6)^6[23x^3 + 21x + 12]$$

Combination of Terms Example Product Rule:

$$\text{Given: } H(x) = x \ln(x) - x$$

$$\text{we will also use: } \frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

$$\text{Term 1: } f(x)g(x) = x \ln(x)$$

$$f(x) = x$$

$$f'(x) = 1$$

$$g(x) = \ln(x)$$

$$g'(x) = \frac{1}{x}$$

$$H'(x) = f(x)g'(x) + g(x)f'(x) \text{ so, we have:}$$

$$H'(x) = x\left(\frac{1}{x}\right) + \ln(x) \cdot (1); \text{ then simplify}$$

$$H'(x) = 1 + \ln(x)$$

$$\text{Term 2: } k(x) = -x$$

$$k(x) = -x$$

$$k'(x) = -1$$

Combining the two pieces, our solution is: $H'(x) = 1 + \ln(x) - 1$; which simplifies to $\ln(x)$.

Algebraic Alternative to Product Rule:

Given: $H(x) = (x^2 + 1)(x^3 + 6)$

One alternate to this is to use algebra and foil the components together before taking the derivative.

$H(x) = (x^2 + 1)(x^3 + 6) = x^5 + 6x^2 + x^3 + 6$; then we can just use the power rule to take the derivative.

$H'(x) = 5x^4 + 12x + 3x^2$

Alternative to Quotient Rule Example:

Given: $H(x) = \frac{x^3+3}{(x+5)^2}$; Keeping in mind the exponent rule: $m^{-a} = \frac{1}{m^a}$, we can rewrite this expression.

$H(x) = (x^3 + 3)(x + 5)^{-2}$

$f(x) = x^3 + 3$
 $f'(x) = 3x^2$

$g(x) = (x + 5)^{-2}$
 $g'(x) = -2(x + 5)^{-3}$

$H'(x) = \frac{d}{dx} [f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$; ; so, we have:

$H'(x) = (x^3 + 3)[-2(x + 5)^{-3}] + (x + 5)^{-2}(3x^2)$; then we begin simplifying if necessary

$H'(x) = (x + 5)^{-3}[-2(x^3 + 3) + (x + 5)(3x^2)]$; factoring out like terms

$H'(x) = (x + 5)^{-3}[(-2x^3 - 6) + (3x^3 + 15x^2)]$; combining like terms

$H'(x) = (x + 5)^{-3}[-6 + 2x^3 + 15x^2]$; one last re-write

$H'(x) = \frac{-6 + 2x^3 + 15x^2}{(x + 5)^3}$

You try's:

1. $H(x) = (x + 5)(x^2 + 1)^6$
2. $H(x) = x^2 e^{-3x}$
3. $H(x) = \sec(\theta) \tan(\theta)$

Solutions:

1. $H'(x) = (x^2 + 1)^5(13x^2 + 60x + 1)$
2. $H'(x) = x e^{-3x}(-3x + 2)$
3. $H'(x) = \sec^3(\theta) + \sec(\theta)\tan^2(\theta)$ or $\sec(\theta)[2\tan^2(\theta) + 1]$